AIEEE - 2008 Mathematics of 27-04-2008

1. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ?
(1) $a = 1, b = 6$ (2) $a = 3, b = 4$ (3) $a = 0, b = 7$ (4) $a = 5, b = 2$

1. **(2)** $(a + b + 23) / 5 = 6 \Rightarrow a + b = 7$
$\sigma^2 = (\sqrt{((1/5)((a-6)^2 + (b-6)^2 + 2^2 + 1^2 + 4^2))})^2 = 6.8$
$\Rightarrow (1/5)[(a-6)^2 + (b-6)^2 + 21] = 6.8$
$\Rightarrow ((a-6)^2 + (b-6)^2 = 13 \Rightarrow a = 3, b = 4.$

2. The vector $\vec{a} = \alpha\vec{i} + 2\vec{j} + \beta\vec{k}$ lies in the plane of the vectors $\vec{b} = \vec{i} + \vec{j}$ and $\vec{c} = \vec{j} + \vec{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ?
(1) $\alpha = 2, \beta = 1$ (2) $\alpha = 1, \beta = 1$ (3) $\alpha = 2, \beta = 2$ (4) $\alpha = 1, \beta = 2$

2. **(2)** $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$
$\Rightarrow (\alpha\vec{i} + 2\vec{j} + \beta\vec{k}) \cdot (\vec{k} - \vec{j} + \vec{i}) = 0$
$\Rightarrow \alpha - 2 + \beta = 0 \Rightarrow \alpha + \beta = 2$
and $\vec{b} \cdot \vec{c} = 1 = \sqrt{2}\sqrt{2}\cos\theta \Rightarrow \theta = 60°$
So, $\alpha + 2 = \sqrt{(\alpha^2 + 4 + \beta^2)}\sqrt{2} \times (\sqrt{3}/2) \Rightarrow \alpha = 1, \beta = 1.$

3. The non zero vector $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is
(1) $\pi / 2$ (2) π (3) 0 (4) $\pi / 4$

3. **(2)** Direction of $\vec{a}$ and $\vec{c}$ are opposite.

4. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point (0, 17 / 2, – 13 / 2). Then
(1) $a = 6, b = 4$ (2) $a = 8, b = 2$ (3) $a = 2, b = 8$ (4) $a = 4, b = 6$

4. **(1)** $\begin{vmatrix} 5 & 1 & a \\ 3 & b & 1 \\ 0 & 17/2 & -13/2 \end{vmatrix} = 0 \Rightarrow 51a - 65b = 46 \quad \therefore \quad a = 6, b = 4.$

5. If the straight lines $\dfrac{x-1}{k}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$ and $\dfrac{x-2}{3}=\dfrac{y-2}{k}=\dfrac{z-1}{2}$ intersect at a point, then the integer k is equal to

(1) 2 (2) – 2 (3) – 5 (4) 5

5. **(3)** Pts. $(\lambda k + 1, 2\lambda + 2, 3\lambda + 3)$ and $(3\mu + 2, k\mu + 3, 2\mu + 1)$

$\Rightarrow \lambda k + 1 = 3\mu + 2, 2\lambda + 2 = k\mu + 3, 3\lambda + 3 = 2\mu + 1 \quad \Rightarrow \quad k = -5.$

6. The differential equation of the family of circles with fixed radius 5 units and centre of the line $y = 2$ is

(1) $(y-2)^2 y'^2 = 25 - (y-2)^2$ (2) $(x-2)^2 y'^2 = 25 - (y-2)^2$

(3) $(x-2) y'^2 = 25 - (y-2)^2$ (4) $(y-2) y'^2 = 25 - (y-2)^2$

6. **(1)** $(x-h)^2 + (y-2)^2 = 5^2$

Differentiating, $2(x-h) + 2(y-2) y' = 0$

$\Rightarrow \quad (y-2) y' + x = h \quad \Rightarrow \quad (y-2)^2 (y')^2 = 25 - (y-2)^2.$

7. Let a, b, c be any real numbers. Suppose that there are nubers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(1) 0 (2) 1 (3) 2 (4) – 1

7. **(2)** $x = cy + bz, y = az + cx, z = bx + ay \quad \Rightarrow \quad \begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$

$\Rightarrow \quad (a^2 + 1) + c(-c - ab) - b(+ca + b) = 0 \Rightarrow a^2 + b^2 + c^2 + 2abc = 1.$

8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true ?

(1) If det $A = \pm 1$, then A^{-1} exists and all its entries are integers.

(2) If det $A = \pm 1$, then A^{-1} need not exist.

(3) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers.

(4) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non integers.

8. **(1)** All co-factors are integers.

9. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 **:** 3. Then the common root is

(1) 3 (2) 2 (3) 1 (4) 4

9. **(2)** $9k + \alpha = 6, 4k\alpha = a$

$3k + \alpha = e, 3k\alpha = 6$

$\Rightarrow \quad k\alpha = 2 \quad \Rightarrow \quad a = 8$

$x^2 - 6x + 8 = 0 \quad \Rightarrow \quad x^2 - 4x - 2x + 8 = 0$

$\Rightarrow \quad (x-4) - 2(x-4) = 0 \quad \Rightarrow \quad x = 2, 4$

$\alpha = 8 / 4k = 2 / k$

$4k + (2 / k) = 6 \quad \Rightarrow \quad 4k^2 - 6k + 2 = 0 \quad \Rightarrow \quad 4k^2 - 4k - 2k + 2 = 0$

$4k(k-1) - 2(k-1) = 0 \Rightarrow \quad k = 1$ (integer)

other roots 4, 3 $\quad \Rightarrow \quad$ common root = 2.

10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent ?

(1) $6 . 8 . {}^7C_4$ (2) $7 . {}^6C_4 . {}^8C_4$ (3) $8 . {}^6C_4 . {}^7C_4$ (4) $7 . 7 . {}^8C_4$

10. **(2)** $M = 1, I = 4, S = 4, P = 2$

No. of ways = $(7! / (4! \times 2!)) \times {}^8C_4$.

11. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true ?

(1) $I < 2 / 3$ and $J > 2$ (2) $I > 2 / 3$ and $J < 2$

(3) $I > 2 / 3$ and $J > 2$ (4) $I < 2 / 3$ and $J < 2$

11. **(4)** ${}_0\int^1 \cos x / x < {}_0\int^1 (1 / \sqrt{x})\, dx = 2$

${}_0\int^1 \sin x / \sqrt{x} < {}_0\int^1 \sqrt{x}\, dx = 2 / 3$.

12. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(1) 2 / 3 (2) 4 / 3 (3) 5 / 3 (4) 1 / 3

12. **(2)** $-2y^2 = 1 - 3y^2 \quad \Rightarrow \quad y = \pm 1$

$x = -2$

${}_{-1}\int^1 (1 - y^2)\, dy = 4 / 3$.

13. The value of $\sqrt{2}\int \frac{\sin x\, dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is

(1) $x + \log | \sin (x - (\pi / 4)) | + c$ (2) $x - \log | \cos (x - (\pi / 4)) | + c$

(3) $x + \log | \cos (x - (\pi / 4)) | + c$ (4) $x - \log | \sin (x - (\pi / 4)) | + c$

13. **(1)** $I = \int \frac{2\sin x\, dx}{\sin x - \cos x} \quad \Rightarrow \quad I = \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x - \cos x)} dx$

$\Rightarrow \quad I = ln | \sin x - \cos x | + x + c$.

14. The statement $p \to (q \to p)$ is equivalent to

(1) $p \to (p \wedge q)$ (2) $p \to (p \leftrightarrow q)$

(3) $p \to (p \to q)$ (4) $p \to (p \vee q)$

14. **(4)** Statement given in (4) is in accordance with given statement.

15. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

(1) 4 / 17 (2) 5 / 17 (3) 6 / 17 (4) 3 / 17

15. **(3)** $\cot\left(\cot^{-1}\frac{4}{3} + \cot^{-1}\frac{3}{2}\right) = \cot\cot^{-1}\frac{\frac{4}{3}\times\frac{3}{2} - 1}{\frac{4}{3} + \frac{3}{2}} = 6 / 17$.

16. Let A be a 2 × 2 matrix with real entries. Let I be 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement - 1 : If $A \neq I$ and $A \neq -I$, then det $A = -1$.

Statement - 2 : If $A \neq I$ and $A \neq -I$, then tr(A) $\neq 0$.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.

(2) Statement - 1 is true, Statement - 2 is false.

(3) Statement - 1 is false, Statement - 2 is true.

(4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

16. **(2)** $\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} a_1^2 + a_2b_1 & a_1a_2 + a_2b_2 \\ a_1b_1 + b_1b_2 & a_2b_1 + b_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a_2\,(a_1 + b_2) = 0,\ b_2\,(a_1 + b_2) = 0 \quad \Rightarrow \quad b_2 = -\,a_1$

$= (-\,a_1^2 + a_2\,b_1) = -\,1.$

17. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is rational number iff y is a transcendental number".

Statement - 1 : r is equivalent to either q or p.

Statement - 2 : r is equivalent to $\sim (p \leftrightarrow -\,q)$.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.

(2) Statement - 1 is true, Statement - 2 is false.

(3) Statement - 1 is false, Statement - 2 is true.

(4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

17. **(3)** Statement 1 is always false.

18. In a shop there are five types of ice-creams available. A child buys six ice creams.

Statement - 1 : The number of different ways the child can buy the six ice creams is ${}^{10}C_5$.

Statement - 2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.

(2) Statement - 1 is true, Statement - 2 is false.

(3) Statement - 1 is false, Statement - 2 is true.

(4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

18. **(3)** $\sum_{i=1}^{6} x_i = 6\,,\ 6 \geq x_i \geq 0$

no. of ways = ${}^{10}C_6$.

19. Statement - 1 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)2^{n-1}$.

Statement - 2 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r\,x^r = (1+x)^n + nx\,(1+x)^{n-1}$

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(2) Statement - 1 is true, Statement - 2 is false.
(3) Statement - 1 is false, Statement - 2 is true.
(4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

19. **(4)** $\frac{d}{dx}\{(1+x)^n.x\} = \frac{d}{dx}\sum_{r=0}^{n}{}^nC_r.x^r$.

20. Statement - 1 : For every natural number $n \geq 2$, $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+....+\frac{1}{\sqrt{n}} > \sqrt{n}$.

Statement - 2 : For every natural number $n \geq 2$, $\sqrt{n(n+1)} < n+1$.

(1) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(2) Statement - 1 is true, Statement - 2 is false.
(3) Statement - 1 is false, Statement - 2 is true.
(4) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

20. **(1)** $\frac{\sqrt{n+1}}{\sqrt{n}} > 1$ $\Rightarrow$ Statement - 2 is correct.

$\Rightarrow$ $\frac{\sqrt{n}}{\sqrt{1}}+\frac{\sqrt{n}}{\sqrt{2}}+....\frac{\sqrt{n}}{\sqrt{n}} > n$ $\Rightarrow$ Statement - 1 is correct.

21. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(1) $\frac{-1}{i+1}$ (2) $\frac{1}{i-1}$ (3) $\frac{-1}{i-1}$ (4) $\frac{1}{i+1}$

21. **(1)** $\frac{i+1}{2} = \frac{-1}{i+1}$.

22. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y) : x - y \text{ is an integer}\}$
Which one of the following is true ?
(1) S is an equivalence relation on R but T is not
(2) T is an equivalence relation on R but S is not
(3) Neither S nor T is an equivalence relation on R
(4) Both S and T are equivalence relations on R

22. **(2)** x change to y, T will not change whereas S will change.

23. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$
where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Shwo that f is invertible and its inverse is

(1) $g(y) = \dfrac{y+3}{4}$ (2) $g(y) = \dfrac{y-3}{4}$

(3) $g(y) = \dfrac{3y+4}{3}$ (4) $g(y) = 4 + \dfrac{y+3}{4}$

23. **(2)** $y = 4x + 3 \quad \Rightarrow \quad x = (y - 3) / 4$.

24. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45°. Then the height of the pole is

(1) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}-1)\,\text{m}$ (2) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}+1}\,\text{m}$

(3) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}-1}\,\text{m}$ (4) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)\,\text{m}$

24. **(4)** $(h / \sqrt{3}) - h = 7 \quad \Rightarrow \quad \dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)\,\text{m}$.

25. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is
(1) 1 (2) 2 / 5 (3) 3 / 5 (4) 0

25. **(1)** $P(A \cup B) = P\{1, 2, 3, 4, 5, 6\} = 1$.

26. It is given that the events A and B are such that $P(A) = 1 / 4$, $P(A \mid B) = 1 / 2$ and $P(B \mid A) = 2 / 3$. Then $P(B)$ is
(1) 2 / 3 (2) 1 / 2 (3) 1 / 6 (4) 1 / 3

26. **(4)** $P(A \cap B) / P(B) = 1 / 2$, $P(A \cap B) / P(A) = 1 / 3$
$\Rightarrow \quad P(A) / P(B) = 3 / 4 \Rightarrow \quad P(B) = 1 / 3$.

27. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $1 / 2$. Then the length of the semi-major axis is

(1) $4 / 3$ (2) $5 / 3$ (3) $8 / 3$ (4) $2 / 3$

27. **(3)** $3a / 2 = 4 \Rightarrow a = 8 / 3$.

28. A parabola has the origin as its focus and the line $x = 2$ as directrix. Then the vertex of the parabola is at

(1) (0, 1) (2) (2, 0) (3) (0, 2) (4) (1, 0)

28. **(4)** Vertex is mid point $\equiv (1, 0)$.

29. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(1) $(-3, -4)$ (2) $(3, 4)$ (3) $(3, -4)$ (4) $(-3, 4)$

29. **(1)** Centre $\equiv (-1, -2)$ mid point of segment. $\Rightarrow$ Point is $(-3, -4)$.

30. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept -4. Then a possible value of k is

(1) -2 (2) -4 (3) 1 (4) 2

30. **(2)** $y + 4 = (-k + 1)(x - 0)$

$\Rightarrow (7 / 2) + 4 = (-1 + k)((1 + k) / 2)$

$\Rightarrow 15 = (-1 + k^2) \Rightarrow k^2 = 16 \Rightarrow k = 4, -4$.

31. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(1) 12 (2) 4 (3) -4 (4) -12

31. **(4)** $a(1 + r) = 12$, $ar^2(1 + r) = 48 \Rightarrow r = -2, a = -12$.

32. Suppose the cubic $x^3 - px + q$ has three distrinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds ?

(1) The cubic has minima at both $\sqrt{(p / 3)}$ and $-\sqrt{(p / 3)}$

(2) The cubic has maxima at both $\sqrt{(p / 3)}$ and $-\sqrt{(p / 3)}$

(3) The cubic has minima at $\sqrt{(p / 3)}$ and maxima at $-\sqrt{(p / 3)}$

(4) The cubic has minima at $-\sqrt{(p / 3)}$ and maxima at $\sqrt{(p / 3)}$

32. **(3)** A cubic with three distinct real roots should have two distinct extremas. Since coeff. of x^3 is +ve first maxima, then minima.

33. How many real solutions does the equation $x^7 + 17x^5 + 16x^3 + 30x - 560 = 0$ have ?

(1) 3 (2) 5 (3) 7 (4) 1

33. **(4)** $f'(x) > 0 \ \forall x \quad \therefore$ Polynomial $f(x)$ has one real root.

34. Let $f(x)=\begin{cases}(x-1)\sin\dfrac{1}{x-1} & \text{if } x\neq 1\\ 0 & \text{if } x=1\end{cases}$

Then which one of the following is true ?

(1) f is differentiable at $x = 0$ but not at $x = 1$

(2) f is differentiable at $x = 1$ but not at $x = 0$

(3) f is neither differentiable at $x = 0$ nor at $x = 1$

(4) f is differentiable at $x = 0$ and at $x = 1$

34. **(1)** R.H.D. = $Lim_{h \to 0} \sin(1/h)/h = \infty$

L.H.D. = $Lim_{h \to 0} -h \sin(-1/h)/-h = -\infty$

$\therefore$ R.H.D $\neq$ L.H.D.; not differentiable.

35. The solution of the differential equation $\dfrac{dy}{dx}=\dfrac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

(1) $y = xe^{(x-1)}$ (2) $y = x\, ln\, x + x$ (3) $y = ln\, x + x$ (4) $y = x\, ln\, x + x^2$

35. **(2)** $(dy/dx) - (y/x) = 1$ I.F. $= e^{-\int (1/x)\, dx}$

$\therefore$ $y/x = \int (1/x)\, dx = ln\, x + c$ $\Rightarrow$ $c = 1$ $\therefore$ $y = x\,(ln\, x + 1)$.