CODE **0**

Note: Code Chart at the end

NARAYANA IIT ACADEMY

presents

IIT-JEE 2007 SOLUTIONS

Physics (Code 0)

PAPER - I

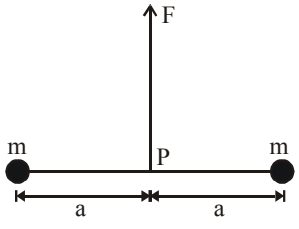
SECTION - I

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A resistance of 2 Ω is connected across one gap of a metre–bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is
   (A) 3 Ω (B) 4 Ω
   (C) 5 Ω (D) 6 Ω.

2. In an experiment to determine the focal length (f) of a concave mirror by the u–v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then,
   (A) $x < f$ (B) $f < x < 2f$
   (C) $x = 2f$ (D) $x > 2f$.

3. Two particles of mass m each are tied at the ends of a light string of length 2a. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the center P (as shown in the figure). Now, the mid–point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes 2x, is



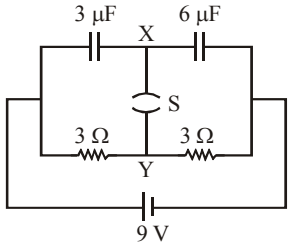
(A) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (B) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(C) $\frac{F}{2m}\frac{x}{a}$ (D) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$.

4. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.
(A) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(B) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(C) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
(D) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

5. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,
(A) Negative and distributed uniformly over the surface of the sphere.
(B) Negative and appears only at the point on the sphere closest to the point charge.
(C) Negative and distributed non–uniformly over the entire surface of the sphere.
(D) Zero.

6. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge the flows from Y to X is
(A) 0 (B) 54 μC
(C) 27 μC (D) 81 μC.



7. A ray of light traveling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be
(A) Only a reflected ray and no refracted ray.
(B) Only a refracted ray and no reflected ray.
(C) A reflected ray and a refracted ray and the angle between them would be less than $180^\circ - 2\theta$.
(D) A reflected ray and a refracted ray and the angle between them would be greater than $180^\circ - 2\theta$.

8. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is
(A) $E\left(^{236}_{92}U\right) > E\left(^{137}_{53}I\right) + E\left(^{97}_{39}Y\right) + 2E(n)$
(B) $E\left(^{236}_{92}U\right) < E\left(^{137}_{53}I\right) + E\left(^{97}_{39}Y\right) + 2E(n)$
(C) $E\left(^{236}_{92}U\right) < E\left(^{140}_{56}Ba\right) + E\left(^{94}_{36}Kr\right) + 2E(n)$

(D) $E\left(^{236}_{92}U\right) = E\left(^{140}_{56}Ba\right) + E\left(^{94}_{36}Kr\right) + 2E(n)$.

9. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(A) 802 nm (B) 823 nm
(C) 1882 nm (D) 1648 nm.

SECTION – II

Assertion – Reason Type

This section contains 4 questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

10. STATEMENT – 1
A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle 30º with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.
because
STATEMENT – 2
The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

11. STATEMENT – 1
In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.
because
STATEMENT – 2
In an elastic collision, the linear momentum of the system is conserved.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

12. STATEMENT – 1
The formula connected u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

because

STATEMENT – 2
Law of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

13. STATEMENT – 1
If the accelerating potential in an X–ray tube is increased, the wavelengths of the characteristic X–ray do not change.
because
STATEMENT – 2
When an electron beam strikes the target in an X–ray tube, part of the kinetic energy is converted into X–ray energy.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

SECTION – III
Linked Comprehension Type

This section contains 2 paragraphs $C_{14\text{-}16}$ and $C_{17\text{-}19}$. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

$P_{14–16}$: Paragraph for Question Nos. 14 to 16

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

14. The radio x_1/x_2 is

(A) 2 (B) $\frac{1}{2}$
(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$.

15. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$

(C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$.

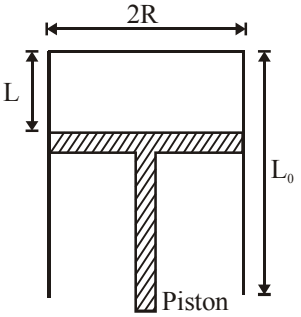
16. The loss of kinetic energy during the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$

(C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$.

P17–19: Paragraph for Question Nos. 17 to 19

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



17. The piston is now pulled out slowly and held at distance 2L from the top. The pressure in the cylinder between its top and the piston will then be

(A) P_0 (B) $\frac{P_0}{2}$

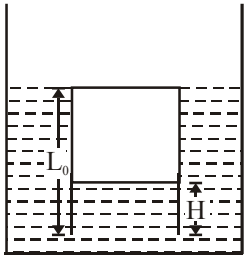
(C) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$.

18. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(A) $\left(\frac{2P_0\pi R^2}{\pi R^2P_0+Mg}\right)(2L)$ (B) $\left(\frac{P_0\pi R^2-Mg}{\pi R^2P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2+Mg}{\pi R^2P_0}\right)(2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2P_0-Mg}\right)(2L)$.

19. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(A) $\rho g(L_0 - H)^2 + P_0 (L_0 - H) + L_0 P_0 = 0$

(B) $\rho g(L_0 - H)^2 - P_0 (L_0 - H) - L_0 P_0 = 0$

(C) $\rho g(L_0 - H)^2 + P_0 (L_0 - H) - L_0 P_0 = 0$

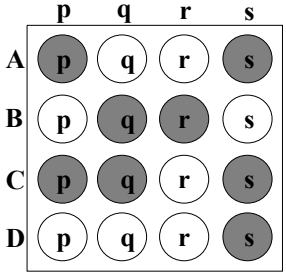
(D) $\rho g(L_0 - H)^2 - P_0 (L_0 - H) + L_0 P_0 = 0$.

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each questions contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



20. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II** and indicate your answer by darkening appropriated bubbles in the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	GM_eM_s G – universal gravitational constant, M_e – mass of the earth, M_s – mass of the sun.	(p)	(volt) (coulomb) (metre)
(B)	$\frac{3RT}{M}$ R – universal gas constant, T – absolute temperature, M – molar mass	(q)	(kilogram) (metre)^3 (second)$^{-2}$
(C)	$\frac{F^2}{q^2B^2}$ F – force, q – charge, B – magnetic field.	(r)	(metre)^2 (second)$^{-2}$
(D)	$\frac{GM_e}{R_e}$ G – universal gravitational constant, M_e – mass of the earth, R_e – radius of earth.	(s)	(farad) (volt)^2 (kg)$^{-1}$

21. **Column I** gives certain situations in which a straight metallic wire of resistance R is used and **Column II** gives some resulting effects. Match the statements in **Column I** with the

statements in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	A charged capacitor is connected to the ends of the wire.	(p)	A constant current flows through the wire.
(B)	The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion.	(q)	Thermal energy is generated in the wire.
(C)	The wire is placed in a constant electric field that has a direction along the length of the wire.	(r)	A constant potential difference develops between the ends of the wire.
(D)	A battery of constant emf is connected to the ends of the wire.	(s)	Charges of constant magnitude appear at the ends of the wire.

22. Some laws / processes are given in **Column I**. Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	Transition between two atomic energy levels.	(p)	Characteristic X–ray.
(B)	Electron emission from a material.	(q)	Photoelectric effect.
(C)	Mosley's law.	(r)	Hydrogen spectrum.
(D)	Change of photon energy into kinetic energy of electrons.	(s)	β – decay.

Chemistry

SECTION – I

Straight Objective Type

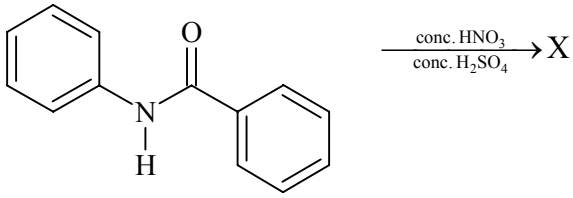
This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

23. The species having bond order different from that in CO is
(A) NO^- (B) NO^+
(C) CN^- (D) N_2

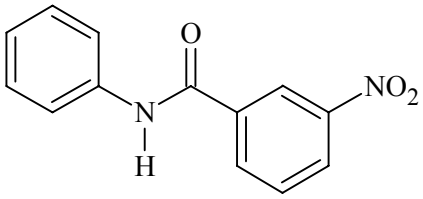
24. Among the following the paramagnetic compound is
(A) Na_2O_2 (B) O_3
(C) N_2O (D) KO_2

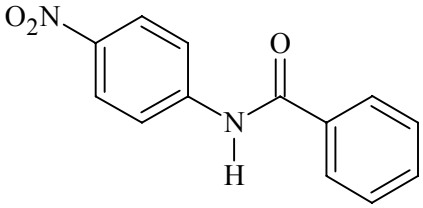
25. Extraction of zinc from zinc blende is achieved by
(A) electrolytic reduction
(B) roasting followed by reduction with carbon
(C) roasting followed by reduction with another metal
(D) roasting followed by self-reduction

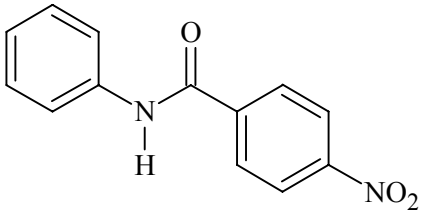
26. In the following reaction,

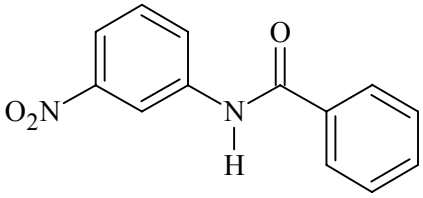


the structure of the major product 'X' is

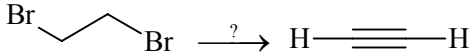
(A) 

(B) 

(C) 

(D) 

27. The reagent(s) for the following conversion,



is/are

(A) alcoholic KOH
(B) alcoholic KOH followed by $NaNH_2$
(C) aqueous KOH followed by $NaNH_2$
(D) Zn/CH_3OH

28. The number of structural isomers for C_6H_{14} is

(A) 3
(B) 4
(C) 5
(D) 6

29. The percentage of p-character in the orbital forming P – P bond in P_4 is

(A) 25
(B) 33
(C) 50
(D) 75

30. When 20 g of naphthoic acid ($C_{11}H_8O_2$) is dissolved in 50 g of benzene (K_f = 1.72 K kg mol^{-1}), a freezing point depression of 2K is observed. The van't Hoff factor (i) is

(A) 0.5
(B) 1
(C) 2
(D) 3

31. The value of $\log_{10}K$ for a reaction $A \rightleftharpoons B$ is

(Given : $\Delta_r H^o_{298K} = -54.07$ kJ mol^{-1}, $\Delta_r S^o_{298K} = 10$ $JK^{-1}mol^{-1}$ and R = 8.314 JK^{-1} mol^{-1}; $2.303 \times 8.314 \times 298 = 5705$)

(A) 5
(B) 10
(C) 95
(D) 100

SECTION – II

Assertion – Reason Type

This section contains 4 questions numbered 32 to 35. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

32. STATEMENT-1 : Boron always forms covalent bond.
because
STATEMENT-2 : The small size of B^{3+} favours formation of covalent bond.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

33. STATEMENT-1 : In water, orthoboric acid behaves as a week monobasic acid.
because
STATEMENT-2 : In water, orthoboric acid acts as a proton donor.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

34. STATEMENT-1: p-Hydroxybenzoic acid has a lower boiling point than o-hydroxybenzoic acid.
because
STATEMENT-2: o-Hydroxybenzoic acid has intramolecular hydrogen bonding.
(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1.
(B) Statement-1 is Ture, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is Fasle, Statement-2 is True

35. STATEMENT-1 : Micelles are formed by surfactant molecules above the critical micellar concentration (CMC).
because
STATEMENT-2 : The conductivity of a solution having surfactant molecules decrease sharply at the CMC.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs C_{36-38} and C_{39-41}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

C_{36-38}: Paragraph for Questions Nos. 36 to 38

The noble gases have closed-shell electronic configuration and are monoatomic gases under normal conditions. The low boiling points of the lighter noble gases are due to weak dispersion forces between the atoms and the absence of other interatomic interactions.

The direct reaction of xenon with fluorine leads to a series of compounds with oxidation numbers +2, +4 and +6. XeF_4 reacts violently with water to give XeO_3. The compounds of xenon exhibit rich stereochemistry and their geometries can be deduced considering the total number of electron pairs in the valence shell.

36. Argon is used in arc welding because of its
(A) low reactivity with metal
(B) ability to lower the melting point of metal
(C) flammability
(D) high calorific value

37. The structure of XeO_3 is
(A) linear
(B) planar
(C) pyramidal
(D) T-shaped

38. XeF_4 and XeF_6 are expected to be
(A) oxidizing
(B) reducing
(C) unreactive
(D) strongly basic

C_{39-41} : Paragraph for Questions Nos. 39 to 41

Chemical reactions involve interaction of atoms and molecules. A large number of atoms/molecules (approximately 6.023×10^{23}) are present in a few grams of any chemical compound varying with their atomic/molecular masses. To handle such large numbers conveniently, the mole concept was introduced. This concept has implications in diverse areas such as analytical chemistry, biochemistry, electrochemistry and radiochemistry. The following example illustrates a typical case, involving chemical/electrochemical reaction, which requires a clear understanding of the mole concept.

A 4.0 molar aqueous solution of NaCl is prepared and 500 mL of this solution is electrolysed. This leads to the evolution of chlorine gas at one of the electrodes (atomic mass: Na = 23, Hg = 200; 1 Faraday = 96500 coulombs).

39. The total number of moles of chlorine gas evolved is
(A) 0.5
(B) 1.0
(C) 2.0
(D) 3.0

40. If the cathode is a Hg electrode, the maximum weight (g) of amalgam formed from this solution is
(A) 200
(B) 225
(C) 400
(D) 446

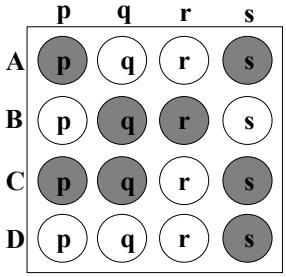
41. The total charge (coulombs) required for complete electrolysis is
(A) 24125 (B) 48250
(C) 96500 (D) 193000

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each questions contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



42. Match the complexes in **Column I** with their properties listed in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	$[Co(NH_3)_4(H_2O)_2]Cl_2$	(p)	geometrical isomers
(B)	$[Pt(NH_3)_2Cl_2]$	(q)	paramagnetic
(C)	$[Co(H_2O)_5Cl]Cl$	(r)	diamagnetic
(D)	$[Ni(H_2O)_6]Cl_2$	(s)	metal ion with +2 oxidation state

43. Match the chemical substances in **Column I** with type of polymers/type of bonds in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	cellulose	(p)	natural polymer
(B)	nylon-6, 6	(q)	synthetic polymer
(C)	protein	(r)	amide linkage
(D)	sucrose	(s)	glycoside linkage

44. Match gases under specified conditions listed in **Column I** with their properties/laws in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	hydrogen gas (P = 200atm, T=273 K)	(p)	compressibility factor ≠ 1
(B)	hydrogen gas (P ~ 0, T = 273 K)	(q)	attractive forces are dominant
(C)	CO_2 (P = 1 atm, T = 273 K)	(r)	PV = nRT
(D)	real gas with very large molar volume	(s)	P(V – nb) = nRT

Mathematics

SECTION – I

This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

45. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx - r = 0$. Then the values of r is

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

46. Let f(x) be differentiable on the interval $(0, \infty)$ such that f(1) = 1, and $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$ for each $x > 0$. Then f(x) is

(A) $\frac{1}{3x}+\frac{2x^2}{3}$ (B) $\frac{-1}{3x}+\frac{4x^2}{3}$

(C) $\frac{-1}{x}+\frac{2}{x^2}$ (D) $\frac{1}{x}$

47. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{5}$ (D) $\frac{1}{5}$

48. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$

(A) on the left of x = c (B) on the right of x = c

(C) at no point (D) at all points

49. $\lim_{x\to\frac{\pi}{4}}\frac{\int_{2}^{\sec^2 x} f(t)\,dt}{x^2-\frac{\pi^2}{16}}$ equals

(A) $\frac{8}{\pi}f(2)$ (B) $\frac{2}{\pi}f(2)$

(C) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (D) 4f(2)

50. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is
(A) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ (B) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$
(C) $x^2\sin^2\theta - y^2 \cos^2\theta = 1$ (D) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

51. The number of distinct real values of λ, for which the vectors $-\lambda^2\,\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is
(A) zero (B) one
(C) two (D) three

52. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is
(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)^{i\pi/4}$
(C) $(4 + 3i)e^{i\pi/4}$ (D) $(3 + 4i)e^{i\pi/4}$

53. The number of solutions of the pair of equations
$2\sin^2\theta - \cos2\theta = 0$
$2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is
(A) zero (B) one
(C) two (D) four

SECTION – II

Assertion – Reason Type

This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

54. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1, 2, ..., n. Let E be any other event with $0 < P(E) < 1$.
STATEMENT- 1 : $P(H_i|E) > P(E \mid H_i) \cdot P(H_i)$ for i = 1, 2, ... , n.
because
STATEMENT 2 : $\sum_{i=1}^{n} P(H_i) = 1$.
(A) STATEMENT -1 is True, Statement -2 is True, Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement-2 is False
(D) Statement -1 is False, Statement-2 is True.

55. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.
STATEMENT -1: The tangents are mutually perpendicular.
because
STATEMENT-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement -1 is False, statement-2 is True.

56. Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.
STATEMENT-1: $\overrightarrow{PQ}\times\left(\overrightarrow{RS}+\overrightarrow{ST}\right)\neq\vec{0}$.

because

STATEMENT – 2: $\overrightarrow{PQ}\times\overrightarrow{RS}=\vec{0}$ and $\overrightarrow{PQ}\times\overrightarrow{ST}\neq\vec{0}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement -1 is False, statement-2 is True.

57. Let F(x) be a indefinite integral of $\sin^2 x$.
STATEMENT -1 : The function F(x) satisfied $F(x + \pi) = F(x)$ for all real x.
because
STATEMENT -2 : $\sin^2 (x + \pi) = \sin^2 x$ for all real x.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement -1 is False, statement-2 is True.

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs $C_{58\text{-}60}$ and $C_{61\text{-}63}$. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

$M_{58\text{-}60}$: Paragraph for Question Nos. 58 to 60

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r – 1). Let
$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ...

58. The sum $V_1 + V_2 + ... + V_n$ is

(A) $\frac{1}{12}$ n(n + 1) $(3n^2 – n + 1)$ (B) $\frac{1}{12}$ n(n + 1) $(3n^2 + n + 2)$

(C) $\frac{1}{2}$ n$(2n^2 – n + 1)$ (D) $\frac{1}{3}$ $(2n^3 – 2n+ 3)$

59. T_r is alwasy
(A) an odd number (B) an even number
(C) a prime number (D) a composite number

60. Which one of the following is a correct statement?
(A) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 5
(B) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 6
(C) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 11
(D) $Q_1 = Q_2 = Q_3 = ...$

$M_{61\text{-}63}$: Paragraph for Question Nos. 61 to 63

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

61. The ratio of the areas of the triangles PQS and PQR is
(A) $1:\sqrt{2}$ (B) 1 : 2
(C) 1 : 4 (D) 1 : 8

62. The radius of the circumcircle of the triangle PRS is
(A) 5 (B) $3\sqrt{3}$
(C) $3\sqrt{2}$ (D) $2\sqrt{3}$

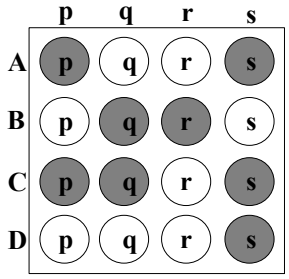
63. The radius of the incircle of the triangle PQR is
(A) 4 (B) 3
(C) $\frac{8}{3}$ (D) 2

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each questions contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



64. Consider the following linear equation
$ax + by + cz = 0$
$bx + cy + az = 0$
$cx + ay + bz = 0$
Match the conditions/expressions in **Column-I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I		**Column II**
(A)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p)	the equations represent planes meeting only at a single point
(B)	$a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q)	the equations represent the line $x = y = z$
(C)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r)	the equations represent identical planes.
(D)	$a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s)	the equations represent the whole of the three dimensional space.

65. In the following [x] denotes the greatest integer less than or equal to x.
Match the functions in **Column I** with the properties in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given the QRS.

	Column I		**Column II**
(A)	$x\|x\|$	(p)	continuous in (-1, 1)
(B)	$\sqrt{\|x\|}$	(q)	differentiable in (-1, 1)
(C)	$x + [x]$	(r)	strictly increasing in (-1, 1)
(D)	$\|x - 1\| + \|x + 1\|$	(s)	not differentiable at least at one point in (-1, 1)

66. Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given the QRS.

	Column I		**Column II**
(A)	$\int_{-1}^{1} \frac{dx}{1+x^2}$	(p)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B)	$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	(q)	$2\log\left(\frac{2}{3}\right)$
(C)	$\int_{2}^{3} \frac{dx}{1-x^2}$	(r)	$\frac{\pi}{3}$
(D)	$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	(s)	$\frac{\pi}{2}$

Answer Key

PHYISCS		CHEMISTRY		MATHEMATICS	
1.	(A)	23.	(A)	45.	(D)
2.	(B)	24.	(D)	46.	(B)
3.	(B)	25.	(B)	47.	(C)
4.	(A)	26.	(B)	48.	(A)
5.	(D)	27.	(B)	49.	(A)
6.	(C)	28.	(C)	50.	(A)
7.	(C)	29.	(D)	51.	(C)
8.	(A)	30.	(A)	52.	(D)
9.	(B)	31.	(B)	53.	(C)
10.	(C)	32.	(A)	54.	(B)
11.	(D)	33.	(C)	55.	(A)
12.	(C)	34.	(D)	56.	(C)
13.	(B)	35.	(B)	57.	(D)
14.	(C)	36.	(A)	58.	(B)
15.	(A)	37.	(C)	59.	(D)
16.	(B)	38.	(A)	60.	(B)
17.	(A)	39.	(B)	61.	(C)
18.	(D)	40.	(D)	62.	(B)
19.	(C)	41.	(D)	63.	(D)
20.	(A) → (p, q)	42.	(A) → (p), (q), (s)	64.	(A) → (r)
	(B) → (r, s)		(B) → (p), (s), (r)		(B) → (q)
	(C) → (r, s)		(C) → (q), (s)		(C) → (p)
	(D) → (r, s)		(D) → (q), (s)		(D) → (s)
21.	(A) → (q)	43.	(A) → (p), (s)	65.	(A) → (p), (q), (r)
	(B) → (r, s)		(B) → (q), (r)		(B) → (p), (s)
	(C) → (s)		(C) → (p), (r)		(C) → (r), (s)
	(D) → (p), (q), (r)		(D) → (s)		(D) → (p), (q)
22.	(A) → (p, (r)	44.	(A) → (p), (s)	66.	(A) → (s)
	(B) → (p), (q), (s);		(B) → (r)		(B) → (s)
	(C) → (p)		(C) → (p), (q)		(C) → (p)
	(D) → (q)		(D) → (p), (q)		(D) → (r)

Solutions

Physics

1. $\frac{2\Omega}{R} = \frac{\ell}{100-\ell}$

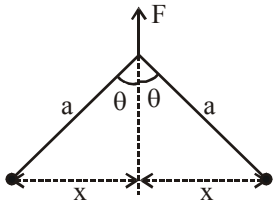
$\frac{R}{2\Omega} = \frac{\ell+20}{100-\ell-20} \Rightarrow R = 3\Omega.$

2. When object is placed between f and 2f real and inverted image is beyond 2f and parallel is found, as described in the question.

3. $\because \quad F = 2T\cos\theta$

$\Rightarrow \quad T = \frac{F}{2\cos\theta}$

Also,



$$\text{acceleration, } f = \frac{T\sin\theta}{m}$$

$$\frac{F}{2m}\tan\theta$$

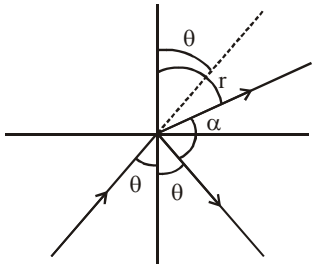
$$= \frac{F}{2m}\frac{x}{\sqrt{a^2 - x^2}}.$$

4. For the electric field in the space between the cylinders, there must be charge inside the outer cylinder.

5. When charge q is placed outside the conducting sphere in order to neutralize the electric field inside the sphere induction takes place. As total charge of an isolated body is conserved, hence net charge on the sphere remains zero.

6. $q_i = 18\ \mu C$
$q_{f(3\mu F)} = 9\ \mu C$
$q_{f(6\mu F)} = 36\ \mu C$
$\Delta q_{(3\mu F)} = -9\mu C - (-18\mu C) = 9\ \mu C$
$\Delta q_{(6\mu F)} = 36\ \mu C - 18\ \mu C = 18\ \mu C$
$\Delta q_{total} = 9\ \mu C + 18\ \mu C = 27\mu C.$

7. In the figure,
$\alpha = \pi - (r + \theta)$
as $r > \theta$
$\Rightarrow\ r + \theta > 2\theta$
$\Rightarrow\ \pi - (r + \theta) < \pi - 2\theta.$



8. $^{236}_{92}U$ is an unstable isotope of Uranium that is formed during the fission of Uranium $\left(^{235}_{92}U\right)$. Since this isotope $\left(^{236}_{92}U\right)$ breaks up into smaller fragment naturally, releasing kinetic energy, the rest mass energy of the initial state is greater than total rest mass energy of the final state.

9. $$K \times \frac{3}{4} = \frac{1}{\lambda_{uv}}$$
$$K \times \frac{1}{9} = \frac{1}{\lambda_{inf}} \Rightarrow \lambda_{inf} = \frac{27}{4} \times \lambda_{uv} \approx 823 \text{ nm}.$$

10. The co-efficient of friction depends on the nature of the contact surfaces. When block moves on the inclined surface, work is done against gravity. Hence gravitational potential energy of the system increases.

11. Momentum of the system of colliding bodies is conserved irrespective of the nature of the collision. Relative speed of bodies means magnitude of the relative velocity of their centres of mass. This does not imply that velocity of approach of points of impact is equal to the velocity of separation of points of impact.

12 Laws are universal, where as mirror formula is based on paraxial approximation.

13. Wavelength of characteristics x-ray depends only on the atomic number of the target element, whereas wavelength of the continuous X-ray depends on that fraction of energy of the incident electron, that gets converted into x-ray photon. These two are independent from each other.

14. $\frac{1}{2}kx_1^2 = \frac{1}{2}I(2\omega)^2$

$\frac{1}{2}kx_2^2 = \frac{1}{2}2I\omega^2$

$\Rightarrow \quad \frac{x_1}{x_2} = \sqrt{2}$

15. From COAM

$4I\omega = 3I\omega' \Rightarrow \omega' = \frac{4\omega}{3}$

$\because \quad$ Impulse $= \tau t = 2I\left(\frac{4\omega}{3} - \omega\right)$

$\Rightarrow \quad \tau = \frac{2I\omega}{3t}$

16. $\Delta KE = \frac{1}{2} \times 3I \times \frac{16\omega^2}{9} - \frac{1}{2}\{2I \times \omega^2 + I \times 4\omega^2\}$

$= \frac{8I\omega^2}{3} - 3I\omega^2 = -\frac{I\omega^2}{3}$

17. Since cylinder is open to atmosphere hence pressure with remain P_0.

18. $P_i \pi R^2 + Mg = P_0 \pi R^2$

$\Rightarrow \quad P_i = P_0 - \frac{Mg}{\pi R^2}$

Also

$P_0 \times 2L\pi R^2 = \left(P_0 - \frac{Mg}{\pi R^2}\right)\ell\pi R^2$

$\Rightarrow \quad \ell = \frac{2P_0L}{P_0 - \frac{Mg}{\pi R^2}}$

19. $P_0L_0A = P(L_0 - H)A \Rightarrow P = \frac{P_0L_0}{(L_0 - H)}$

Also,

$P = P_0 + (L_0 - H)\rho g \Rightarrow P_0L_0 = P_0(L_0 - H) + (L_0 - H)^2 \rho g$

20. (A) – (p, q)
(B) – (r, s)
(C) – (r, s)
(D) – (r, s)

21. (A) – (q)
(B) – (r, s)
(C) – (s)
(D) – (p, q, r)

22. (A) – (p, r)
(B) – (p, q, s)
(C) – (p)
(D) – (q)

Chemistry

SECTION – I

23. **(A)** CO $\quad \sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\pi_{2py}^2\pi_{2pz}^2\pi_{2px}^2$

BO $= \frac{1}{2}[N_b - N_a] = \frac{10-4}{2} = 3$

NO^- $\quad \sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\pi_{2py}^2\pi_{2pz}^2\pi_{2px}^2\pi_{2py}^{*1}$

BO $= \frac{10-5}{2} = 2.5$

24. **(D)** KO_2 contains O_2^- (superperoxide ion) which contains one unpaired electron in π_{2p}^* MO

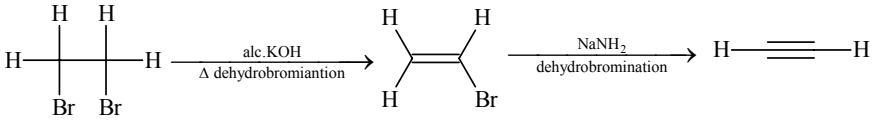
O_2^- $\quad \sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2px}^2\pi_{2py}^2\pi_{2pz}^2\pi_{2px}^{*2}\pi_{2pz}^{*1}$

25. **(B)** $2ZnS + 3O_2 \xrightarrow{\Delta} 2ZnO + 2SO_2\uparrow$

$ZnO + C \xrightarrow{\Delta} Zn + CO\uparrow$

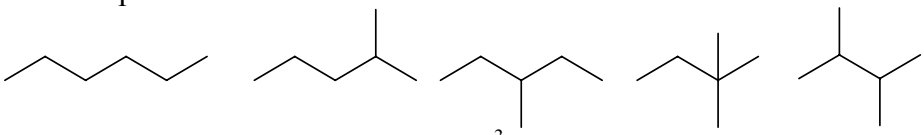
26. **(B)** The ring linked with N atom of $-NH-\overset{\overset{O}{\|}}{C}-$ group is activated while the other linked with carbonyl carbon will be deactivated. The electrophilic substitution will be preferred in activated ring and that to a p-position for least steric repulsion.

27. **(B)**



In second step, alc. KOH can't be used as it is not as strong base as is needed to abstract proton linked with vinylic carbon. $NaNH_2$ is stronger bases. Aqueous KOH on the other hand will effect substitutioin by S_N2 pathway rather than elimination by E2 pathway.

28. **(C)** The five possible structural isomers are as follows:



29. **(D)** The p-character is 75% assuming sp^3 hybridisation of p-atom in P_4. However, the bond angle of 60° is indicative of the bonding though pure p-orbital and then the p-character must be more than 75%. However, there being no such choice given, the correct option is (D)

30. **(D)** MW of napthoic acid (calculated) $= \frac{1000K_f w}{W\Delta T_f}$

$= \frac{1000 \times 1.72 \times 20}{100}$

$= 344$

Actual MW of napththoic acid $= 172$

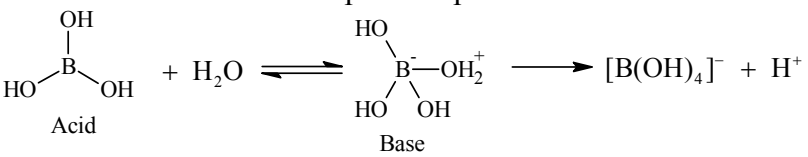
Van't Hoff factor (i) $= \frac{\text{Actual MW}}{\text{Calculated MW}} = \frac{172}{344} = 0.5$

31. **(B)** $\Delta G^0 = \Delta H^0 - T\Delta S^0$
$= -54.07 \times 10^3 - 298 \times 10$
$= -54070 - 2980$
$= -57050$
$\Delta G^0 = -2.303\ RT \log K$
$57050 = 2.303 \times 8.314 \times 298 \log_{10} K$
$\log_{10} K = \dfrac{57050}{2.303 \times 8,.314 \times 298} = 10$

SECTION – II

32. **(A)** B^{3+} has very small size and for having very high charge, it is highly polarizing. So, according to Fajan's rule compounds of boron will be always covalent. In other words, due to very small size of B atom, the sum of its 1st, 2nd and 3rd I.P. will be too high to favour the formation of B^{3+} ion.

33. **(C)** Orthoboric acid (H_3BO_3) is mono basic Lewis acid due to one vacant p-orbital in the central boron atom. It accepts lone pair of solvent water molecule as shown below:



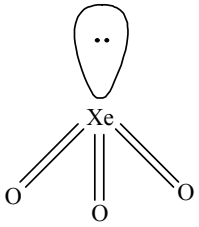
34. **(D)** p-Hydroxybenzoic acid having intermolecular H – bonding has higher boiling point than that of O-hydroxybenzoic acid

35. **(B)**

SECTION – III

36. **(A)** Argon being inert creates inert atmosphere and thereby prevents metal to combine with O_2 of air.

37. **(C)**



38. **(A)**

39. **(B)** No. of moles of NaCl in solution $= 4 \times \dfrac{1}{2} = 2$

$\underset{2\text{ moles}}{2NaCl} \longrightarrow \underset{1\text{ moles}}{Cl_2}$

40. **(D)** $2NaCl \longrightarrow 2Na$

Na	+ Hg ⟶	Na(Hg)
	sodium amalgam	
2 moles	2 moles	2 moles

$= 2 \times (23 + 200)$
$= 446$ g

41. **(D)** $2Cl^- \longrightarrow Cl_2 + 2e$
2×96500
$= 193000C$

SECTION – IV

42. (A) → (p), (q), (s)
(B) → (p), (s), (r)
(C) → (q), (s)
(D) → (q), (s)

43. (A) → (p), (s)
(B) → (q), (r)
(C) → (p), (r)
(D) → (s)

44. (A) → (p), (s)
(B) → (r)
(C) → (p), (q)
(D) → (p), (q)

Mathematics

45. **(D)** $\alpha^2 - p\alpha + r = 0$

$$\frac{\alpha^2}{4} - \frac{q\alpha}{2} + r = 0$$

$$\alpha^2 - p\alpha = \frac{\alpha^2}{4} - \frac{q\alpha}{2}$$

$$\frac{3\alpha^2}{4} = \alpha\left(p - \frac{q}{2}\right)$$

$$\alpha = \frac{4}{3}\left(p - \frac{q}{2}\right)$$

$$\beta = p - \frac{4}{3}\left(p - \frac{q}{2}\right) = p - \frac{4p}{3} + \frac{2q}{3} = \frac{2q}{3} - \frac{p}{3} = \left(\frac{2q - p}{3}\right)$$

$$r = \frac{2}{9}\ (2p - q)\ (2q - p).$$

46. **(B)** $\lim\limits_{t \to x} \dfrac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

$2t\ f(x) - x^2\ f'(t) = 1$

$2x\ f(x) - x^2\ f'(x) = 1$

$$f'(x) - \frac{2}{x}\ f(x) = \frac{1}{x^2}$$

$$I.\ f = e^{\int -\frac{2}{x} dx} = e^{-2\log x} = \frac{1}{x^2}$$

$$\frac{y}{x^2}=\int\frac{1}{x^4}dx+c$$

$$\frac{y}{x^2}=\frac{x^{-3}}{-3}+c$$

$$\frac{y}{x^2}+\frac{1}{3x^2}=c$$

$\because$ f(1) = 1 $\Rightarrow$ $1+\frac{1}{3}=c$

$$c=\frac{4}{3}$$

$$\Rightarrow f(x)=\left(\frac{4}{3}-\frac{1}{3x^3}\right)x^2=\frac{4x^2}{3}-\frac{1}{3x}$$

47. **(C)** $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{4!(2!)^5}{5!(2!)^4}=\frac{2}{5}$

48. **(A)** Slope of tangent at x = c is e^c

Slope of chord = $\frac{e^c(e^2-1)}{2e}>e^c$

Hence the point of intersection in the left of x = c.

49. **(A)** $\lim\limits_{x\to\pi/4}\frac{\int_2^{\sec^2 x}f(t)\,dt}{x^2-\frac{\pi^2}{16}}=\lim\limits_{x\to\pi/4}\frac{f(\sec^2 x).2\sec^2 x\tan x}{2x}$

$=\frac{f(2)\times 2}{\pi/4}=\frac{8}{\pi}f(2)$.

50. **(A)** Let the equation of hyperbola be $\frac{x^2}{a^2}\frac{-y^2}{b^2}=1$

given that ae = 1 & a = sinθ

$\Rightarrow$ e = cosecθ

$b^2 = \sin^2\theta\,(\operatorname{cosec}^2\theta - 1)$

$= \cos^2\theta$

equation is $\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$

$x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$.

51. **(C)** $-\lambda^2 i + j + k,\ i - \lambda^2 j + k,\ i + j - \lambda^2 k$

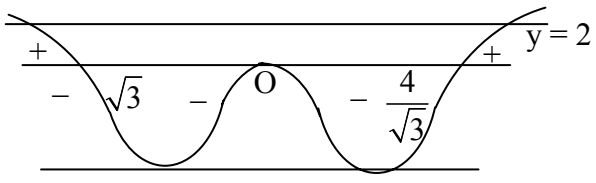
$$\begin{vmatrix}-\lambda^2 & 1 & 1\\ 1 & -\lambda^2 & 1\\ 1 & 1 & -\lambda^2\end{vmatrix}=0$$

$-\lambda^2[\lambda^4-1]-1[-\lambda^2-1]+1[1+\lambda^2]=0$

$-\lambda^6+\lambda^2+\lambda^2+1+1+\lambda^2=0$

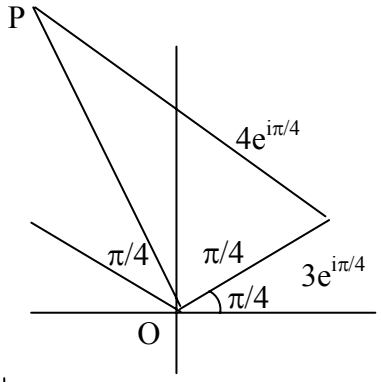
$-\lambda^6+3\lambda^2+2=0$

$-\lambda^6-3\lambda^2-2=0$

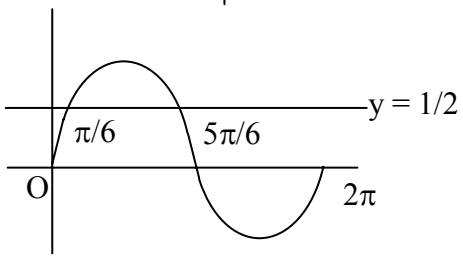


$\lambda^6 = 3\lambda^2 + 2$
$\lambda^6 - 3\lambda^2 = 2$
$\lambda^2 [\lambda^4 - 3] = 2$
$y = \lambda^2 [\lambda^4 - 3] = 0$
$\lambda^2 [\lambda^2 - \sqrt{3}\,] [\lambda^2 + \sqrt{3}\,] = 0$

52. **(D)** $3e^{i\pi/4} + 4e^{3i\pi/4}$
$3e^{i\pi/4} + 4e^{i\pi/2} \cdot e^{i\pi/4}$
$(3 + 4i)\, e^{i\pi/4}$



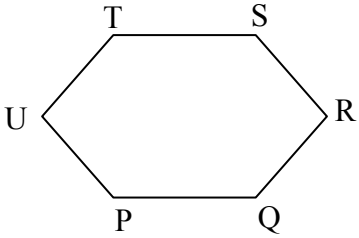
53. **(C)** $2\sin^2\theta - 1 + 2\sin^2\theta = 0$
$\sin^2\theta = 1/4$
$\sin\theta = \pm 1/2$
$2 - 2\sin^2\theta - 3\sin\theta = 0$
$2\sin^2\theta + 3\sin\theta - 2 = 0$
$2\sin^2\theta + 4\sin\theta - \sin\theta - 2 = 0$
$2\sin\theta(\sin\theta + 2) - 1(\sin\theta + 1) = 0$
$\sin\theta = 1/2$



54. **(B)** $P(Hi/E) > P(E \cap H_i)$
$P(Hi/E) = \dfrac{P(E \cap H_i)}{P(E)}$
$0 < P(E) < 1$
$\Rightarrow P(Hi/E) > P(E/Hi)\ PHi$

55. **(A)** Statement–1 is True, statement–2 is True; Statement–2 is a correct explanation for Statement–1.

56. **(C)** $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST})$
$= \overrightarrow{PQ} \times (\overrightarrow{TR}) \neq 0$



57. **(D)** Statement –1 is False, Statement –2 is True

58. **(B)** $a = r,\ d = 2r - 1$

$$Vr = \frac{r}{2}\,[2r + (r-1)(2r-1)]$$

$$= \frac{r}{2}\,[2r + 2r^2 - 3r + 1]$$

$$= \frac{r}{2}\,[2r^2 - r + 1]$$

$$Vr = \frac{1}{2}\,[2r^3 - r^2 + r]$$

$$\sum_{r=1}^{n} Vr = \frac{1}{2}\left[\frac{2n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{4}\,[n(n+1) - \frac{2n+1}{3} + 1]$$

$$= \frac{n(n+1)}{12}\,[3n^2 + 3n - 2n - 1 + 3]$$

$$= \frac{n(n+1)[3n^2 + n + 2]}{12}$$

59. **(D)** $Tr = \frac{1}{2}\,[2(r+1)^3 - (r+1) + (r+1)]$

$$-\frac{1}{2}\,[2r^3 - r^2 + r] - 2\,[-r^2 - 1 - 2r + r^2]$$

$T_r = (3r - 1)(r + 1)$ which is composite number.

60. **(B)** $Q_r = T_{r+1} - T_r$

$= (3r + 2)(r + 2) - (3r - 1)(r + 1)$

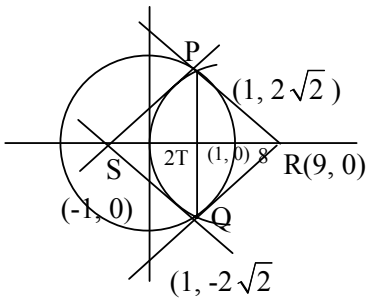
$= 6r + 5$

$\theta_1, \theta_2, \theta_3, \ldots$

11, 17, 23 .... HP with CD – 6.

61 **(C)** Solving $x^2 + y^2 = 9$ & $y^2 = 8x$ We get

$\left(12, \sqrt{2}\right)$ and $(1 - 2\sqrt{2})$



Area of triangle PQR = $\frac{1}{2} \times 4\sqrt{2} \times 8 = 16\sqrt{2}$

Area of triangle PQS = $\frac{1}{2} \times 4\sqrt{2} \times 2 = 4\sqrt{2}$

Area of triangle PQS : PQR = $4\sqrt{2} : 16\sqrt{2} = 1 : 4$

62. **(B)** RS = 10, PR = $\sqrt{64+8} = \sqrt{72}$,

$PS = \sqrt{4+8} = \sqrt{12}$

$= 6\sqrt{2}$

PS = $\sqrt{12}$

Area of triangle PSR = $\frac{1}{2} \times 10 \times 2\sqrt{2} = 10\sqrt{2}$

Circum radius R = $\frac{10 \times 6\sqrt{2} \times 2\sqrt{3}}{10\sqrt{2}} = \frac{12\sqrt{3}}{4} = 3\sqrt{3}$

63. **(D)** Area of the triangle PQR = $16\sqrt{2}$

RP = $6\sqrt{2}$ = RQ

Hence inradius of the triangle

$= \frac{16\sqrt{2} \times 2}{12\sqrt{2} + 4\sqrt{2}} = \frac{16\sqrt{2} \times 2}{16\sqrt{2}} = 1 \times 2 = 2.$

64. **(A–r, B–q, C–p, D–s)**

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc) = -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$= -\ 1/2\ (a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

(A) $\Delta = 0$ and $a = b = c \neq 0$. Identical equations (planes)

(B) $\Delta = 0$ but a, b, c are not all equal. Infinite solutions but planes are different

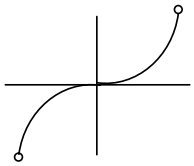
a+b+c=0. So planes are passing through (0, 0, 0) and (1,1, 1) So contain x = y = z line.

(C) $\Delta \neq 0$. Unique solution

(D) $\Delta = a = b = c = 0$. Identities. True for for all values of x, y, z.

65. **(A–p, A–q, A–r; B–p, B–s, C–r, C–s, D–p, D – q)**

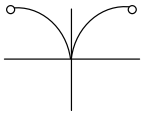
(A) $f(x) = x|x| = \begin{cases} -x^2, & x < 0 \\ x^2, & x > 0 \end{cases}$ is continuous and differentiable everywhere. Also increasing.



(B) $f(x) = \sqrt{|x|} = \begin{cases} \sqrt{-x}, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$

$$f'(x) = \begin{cases} -\dfrac{1}{2\sqrt{-x}}, & x < 0 \\ \dfrac{1}{2\sqrt{x}}, & x > 0 \end{cases}$$

Continuous everywhere differentiable everywhere except at x = 0. increasing.



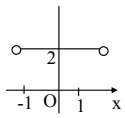
(C) $f(x) = x + [x]$

At integral point x = I,

LHL = I + (I – 1) = 2I – 1, RHL = I + I = 2I = f(I)

So not continuous hence not differentiable at integral points but increasing.

(D) $f(x) = |x - 1| + |x + 1| = \begin{cases} -2x, & x < -1 \\ 2, & -1 \leq x < 1 \\ 2x, & 1 \leq x \end{cases}$



Continuous everywhere, differentiable everywhere except at x = –1 and x = 1 not increasing in (–1,1)

66. **(A–s, B–s, C–p, D–r)**

(A) $\int_{-1}^{1} \frac{dx}{1+x^2} = 2\int_{0}^{1} \frac{dx}{1+x^2} = 2\tan^{-1} x\Big|_0^1 = 2\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{2}$

(B) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x\Big|_0^1 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

(C) $\int_{2}^{3} \frac{dx}{1-x^2} = \frac{1}{2}\left|\frac{1+x}{1-x}\right|\Big|_2^3 = \frac{1}{2}\ln\left(\frac{2}{3}\right)$

(D) $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x\Big|_1^2 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$

CODE CHART

CODE WISE SHUFFLING OF QUESTIONS [IIT 0 7] [PAPER - 1] [CODE: IIT-07]

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Note: However, verify with original paper and zero code paper