SECTION - I

The explanatory answers are based on the question and option order in Set 333. Please refer to the table for tracking the questions in other sets.

1. $\frac{1}{m} = \frac{1}{12} - \frac{4}{n}$

$\Rightarrow m = \frac{12n}{n-48}$

Since m is positive, n must be greater than 48. Possible odd values of n such that 48 < n < 60 are 49, 51, 53, 55, 57 and 59 of which only 49, 51 and 57 give integral values of m.

Hence, [3].

2. Let the original amount be Rs.x and y paise. Then interchanged amount = Rs.y and x paise

From the given condition

$3(100x + y) = 100y + x - 50$

$\Rightarrow 300x + 3y = 100y + x - 50$

$\Rightarrow 299x = 97y - 50$

$\Rightarrow y = \frac{299x + 50}{97}$

Considering the options, only for x = 18, we get an integral value for y i.e., y = 56.

Hence, [2].

3. Possible combinations

Case 1 : Bill is paid with only two 50's misos

$2 \times 50 + 1 \times 10 + 7 \times 1$ ... 1 way

Case 2 : Bill is paid with only one 50 miso

$1 \times 50 + 5 \times 10 + 7 \times 1$

$1 \times 50 + 4 \times 10 + 17 \times 1$

$1 \times 50 + 3 \times 10 + 27 \times 1$

. . .

. . .

. . .

$1 \times 50 + 0 \times 10 + 57 \times 1$

} i.e., 6 ways

Case 3 : Bill is paid with no 50 misos

$10 \times 10 + 7 \times 1$

$9 \times 10 + 17 \times 1$

. .

. .

. .

$1 \times 10 + 97 \times 1$

} i.e., 10 ways

Case 4 : Bill is paid with no 10 misos and 50 misos

1×107 ... 1 way

$\therefore$ Total number of ways = 1 + 6 + 10 + 1 = 18 ways.

Hence, [1].

4.

Quantity produced	CP	SP	Profit
x	$240 + bx + cx^2$	30x	$30x - 240 - bx - cx^2$
20	240 + 20b + 400c	600	600 – 240 – 20b – 400c
40	240 + 40b + 1600c	1200	1200 – 240 – 40b – 1600c
60	240 + 60b + 3600c	1800	1800 – 240 – 60b + 3600c

From the given conditions,

$(240 + 40b + 1600c) = \frac{5}{3}(240 + 20b + 400c)$ ... (i)

Also $240 + 60b + 3600c = \frac{3}{2}(240 + 40b + 1600c)$

$= \frac{5}{2}(240 + 20b + 400c)$ ... (ii)

From (i) $2800c + 20b - 480 = 0$ ... (iii)

$5200c + 20b - 720 = 0$ ... (iv)

$2400c = 240$

$c = \frac{1}{10} \Rightarrow b = 10$

Profit on x units = $f(x) = 30x - 240 - 10x - \frac{x^2}{10}$

i.e., $f(x) = -\frac{x^2}{10} + 20x - 240$

f(x) is maximum at x if $f'(x) = 0$

i.e., $-\frac{2x}{10} + 20 = 0$

$2x = 200$

$x = 100$

Hence, [4].

5. Maximum daily profit = f(100)

$= -1000 + 2000 - 240$

= Rs.760

Hence, [2].

Answers to 6 and 7:

$a_1 = p,\ b_1 = q$

$n = 2 : a_2 = pb_1 = pq\ ;\ b_2 = q\ b_1 = q^2$

$n = 3 : a_3 = pq_2 = p^2q\ ;\ b_3 = qa_2 = pq^2$

$n = 4 : a_4 = pb_3 = p^2q^2\ ;\ b_4 = qb_3 = pq^3$

$n = 5 : a_5 = pa_4 = p^3q^2\ ;\ b_5 = qa_4 = p^2q^3$

$n = 6 : a_6 = pb_5 = p^3q^3\ ;\ b_6 = qb_2 = p^2q^4$

$n = 7 : a_7 = pa_6 = p^4q^3\ ;\ b_7 = qa_6 = p^3q^4$

6. $a_2 + b_2 = pq + q^2 = q(p + q)$

$a_4 + b_4 = p^2q^2 + pq^3 = pq^2(p + q) = q(pq)(p + q)$

$a_6 + b_6 = p^3q^3 + p^2q^4 = p^2q^3(p + q) = q(pq)^2(p + q)$

$\therefore$ In general, $a_n + b_n = q(pq)^{\frac{1}{2}n-1}(p + q)$

Hence, [4].

7. $p = \frac{1}{3},\ q = \frac{2}{3} \Rightarrow p + q = 1$ and $pq = \frac{2}{9}$

i.e., $a_1 + b_1 = 1$

Now, $a_3 + b_3 = pq(p + q) = pq$

$a_5 + b_5 = (pq)^2\ (p + q) = (pq)^2$

$a_7 + b_7 = (pq)^3\ (p + q) = (pq)^3$

In general, for odd 'n' and $p = \frac{1}{3}$, $q = \frac{2}{3}$

$$a_n + b_n = (pq)^{\frac{(n-1)}{2}} = \left(\frac{2}{9}\right)^{\left(\frac{n-1}{2}\right)}$$

Starting from the smallest option

$$a_7 + b_7 = \left(\frac{2}{9}\right)^3 = 0.01$$

$$a_9 + b_9 = \left(\frac{2}{9}\right)^4 = 0.002 < 0.01$$

Hence, [2].

8. Total number of teams = n
Number of players in each team = k
Number of players common to two teams = Number of teams = n
$\therefore$ Total number of players participating in the tournament
= nk – n = n(k – 1)
Hence, [4].

9. Let the four-digit number be
1000a + 100a + 10b + b = 1100a + 11b
This number will be a perfect square if
$1100a + 11b = k^2$; where k is an integer.
$\Rightarrow 11(100a + b) = k^2$

$$\Rightarrow 100a + b = \frac{k^2}{11}$$

$\therefore$ k should be a multiple of 11 such that 100a + b is a 3-digit number k = 44, 55, 66, ...,99
Corresponding values of 100a + b will be 176, 275, 396, 539, 704, 891.
Now, since a and b are digits of a number.
$\therefore$ a, b < 9 only '704' satisfies this.
$\therefore 1100a + 11b = 7700 + 44 = 7744 = (88)^2$
Hence, [3].

10. Let the amount invested in options B and C be in the ratio 1 : K

$\therefore$ Depending on whether there is a rise or fall in the stack market,

the amount earned will be $5 - \frac{5k}{2}$ or $2k - 3$.

$\therefore$ Guaranteed return = min $\left\{\frac{5-5k}{2}, 2k-3\right\}$

$\therefore$ The maximum guaranteed return will be earned when $\frac{5-5k}{2}$ = 2k – 3. i.e., 9k = 16,

i.e., $k = \frac{16}{9}$.

$\therefore$ The maximum guaranteed return is when,

the amounts invested are in the ratio 9 : 16 i.e., 36% and 64% respectively.

Now, the guaranteed return for this distribution is 0.2% (see expans of Q.11)

Since option A gives a return of 0.1% which is lesser than this, no amount should be invested in option A.

$\therefore$ Maximum guaranteed return = 0.20%

Hence, [1].

11. Let us assume that Shabnam has Rs.100.

We calculate her guaranteed return in case of each of the given options:

Option [4] : 0.1% of 100 = 0.1

Option [5] : If there is a rise in the stock market, earning = 5% of 36 – 2.5% of 64

$$= \frac{180-160}{100} = 0.4$$

If there is a fall in the stock market, earning = –3% of 36 + 2% of 64

$$= \frac{-108+128}{100} = 0.2$$

$\therefore$ Guaranteed return = 0.2

Option [1] : Rise in market gives earning

= 5% if 64 + 2.5% of 36 = 2.4

Fall in market gives earning = –3% of 64 + 2% of 36 = –1.2 which is negative.

Similarly, the guaranteed return for options [2] and [3] is also negative.

$\therefore$ Option [5] offers the highest guaranteed return.

Hence, [5].

12. A member (a, b) will have an enemy of the form (c, d) where c, d, a, b are all distinct.

$\therefore$ c and d can be chosen in ${}^{n-2}C_2$ ways.

So, number of enemies = $\frac{(n-2)(n-3)}{2} = \frac{1}{2}(n^2 - 5n + 6)$

Hence, [2].

13. Consider friends (a, b) and (a, c).

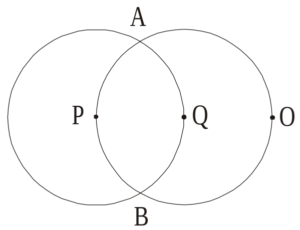
Their common friend can be either (b, c) or a member of the form (a, d) or (d, a) where d is different from a, b, c.

Now, d can be chosen in (n – 3) ways.

$\therefore$ Number of common friends = (n – 3) + 1 = n – 2.

Hence, [2].

14. P and Q do not lie within intersection of the circles. The extreme case can be that they lie on the circumference of the other circle as shown in the figure.



In this case ΔAPQ will be an equilateral triangle.

$\therefore$ $m\angle AQP = 60°$

If $m\angle AQP$ is more than 60°, then P and Q will lie within the intersection of the circles

Hence, [1].

15. From 100 + 0.10n = 89 + 0.15n, n will be definitely greater than 100

Price of Darjeeling tea on 100th day and onwards

= 100 + 0.10 (100)

= Rs.110

Now, price of Ooty tea will be Rs.110

when 89 + 0.15n = 110

$\Rightarrow$ 0.15n = 21

$\Rightarrow$ n = 140

$\therefore$ Prices will be equal on 140th day i.e., May 20.

Hence, [1].

16. Let $f(x) = ax^2 + bx + c$

f attains a maximum at $x = 1$

$f'(x) = 0$

$\Rightarrow 2ax + b = 0$

$\Rightarrow x = \frac{-b}{2a} = 1$

$\Rightarrow -b = 2a$

Also max $f(x) = 3$

$\Rightarrow a + b + c = 3$

$\Rightarrow a - 2a + c = 3$

$\Rightarrow c - a = 3$

$f(0) = 1$

$\Rightarrow c = 1$

$\Rightarrow a = -2$

$\therefore f(x) = -2x^2 + 4x + 1$

$f(10) = -2(100) + 4(10) + 1$

$= -200 + 41 = -159$

Hence, [5].

Answers to questions 17 and 18:

Let the speed of the plane be x km/hr

Then its speed from B to A = (x + 50)km/hr

and its speed from A to B = (x – 50)km/hr

The flight starts from city B(8.00 a.m.) and arrives at city B (8.00 a.m.) after halting for 1 hour in city A

$\therefore$ Total time taken = 11 hours

i.e., $\frac{3000}{x+50} + \frac{3000}{x-50} = 11$

$\Rightarrow \frac{2x}{x^2-2500} = \frac{11}{3000}$

$\Rightarrow 11x^2 - 6000x - 27500 = 0$

Solving the above we get, $x = 550$

$\therefore$ Time taken from B to A = $\frac{3000}{500} = 6$

Flight reaches A when local time in B is 2.00 p.m. which is same as local time 3.00 p.m. in A

$\therefore$ Required time difference = 1 hour

17-[2] 18-[4].

19. $f(n) = \dfrac{f(1)+f(2)+....+f(n-1)}{n^2-1}$

$f(1) = 3600$

$f(2) = \dfrac{f(1)}{3} = \dfrac{3600}{3} = 1200$

$f(3) = \dfrac{f(1)+f(2)}{8} = \dfrac{4800}{8} = 600$

$f(4) = \dfrac{f(1)+f(2)+f(3)}{15} = \dfrac{5400}{15} = 360$

$f(5) = \dfrac{f(1)+....+f(4)}{24} = \dfrac{5760}{24} = 240$

$f(6) = \dfrac{f(1)+....+f(5)}{35} = \dfrac{6000}{35} = \dfrac{1200}{7}$

$f(7) = \dfrac{f(1)+....+f(6)}{48}$

$= \dfrac{6000}{48} + \dfrac{1200}{7\times 48}$

$= 125 + \dfrac{25}{7}$

$f(8) = \dfrac{f(1)+....f(7)}{63} = \dfrac{6125}{63} + \dfrac{1225}{7\times 63} = 100$

$f(9) = \dfrac{f(1)+....+f(8)}{80} = \dfrac{6125+175+100}{80} = 80$

Hence, [4].

20. S = {2, 3, 4, ... 2n + 1}

Total number of elements in S = 2n

$X = \dfrac{3+5+...+(2n+1)}{n}$

$$Y = \frac{2+4+6...+2n}{n}$$

$$\therefore\ X - Y = \frac{(3-2)+(5-4)....+(2n+1-2n)}{n}$$

$$= \frac{1+1+....+1(n\ \text{times})}{n} = \frac{n}{n} = 1$$

Hence, [5].

21. Ten years ago, total age of 8 members = 231

Three years later, sum of the ages = 231 + 8 × 3 – 60 = 195

Three more years later, sum of the ages = 195 + 8 × 3 – 60
= 159

Now, sum of current ages = 159 + 8 × 4 = 191

$\therefore$ Required average = $\frac{191}{8} \approx$ 24 years.

Hence, [3].

22. Using Statement A:

For minimum diameter i.e., 8m,

the capacity of the tank = $\frac{4}{3} \times \frac{22}{7} \times 4^3\ m^3$

= 268.19kl

< 400kl

For diameter greater than 8m, say 9.9m,

the capacity of the tank = $\frac{4}{3} \times \frac{22}{7} \times (4.95)^3\ m^3$

= 508.25 kl

> 400kl

Hence, statement A alone cannot be used to answer the question.

Using statement B:

Volume of material used = $\frac{\text{Mass}}{\text{Density}}$ = V (say), which is given

$\therefore$ Outer volume – Inner volume = V

$\therefore$ We can find the inner volumer of the answer the question. Hence, [2].

23. Using statement A:

$x + y + z = 89$

For $x^2 + y^2 + z^2$ to be minumum, each of x, y, z must take integral value nearest to $\frac{89}{3}$

Let $x = 30,\ y = 30,\ z = 29$

$\therefore$ Minimum value of $x^2 + y^2 + z^2 = (30)^2 + (30)^2 + (29)^2 = 2641$

Thus statement A alone is sufficient to answer the question. Hence, [1].

24. $W_I \equiv$ Average weight of Section I

$W_{II} \equiv$ Average weight of section II

$W_I + W_{II} = 90$ where $W_I < W_{II}$

Let weight of Deepak and Ponam be D and P kgs respectively

Then $\frac{50 \times W_I + D - P}{50} = W_{II}$ and $\frac{50 \times W_{II} - D + P}{50} = W_I$

$\Rightarrow 50(W_{II} - W_I) = D - P$

Using Statement A alone:

$50 \times 1 = D - P$ .... (i)

Thus D and P can take various values

So, Statement A alone is not sufficient.

Using Statement B alone:

$\frac{50 \times W_I + D}{51} = \frac{50W_{II} - D}{49}$ .... (ii)

Since values of W_I and W_{II} are not known

We cannot find the value of D

Combining both the statements,

values of W_I and W_{II} can be found and hence value of D and P can be found, using (i) and (ii).

Hence, [3].

25. The farthest point from point M which is on the square is the diagonally opposite point.

$\therefore\ \ell(OM) \le \sqrt{2} \times$ side of the square.

$\therefore\ \ell(OM) \le \sqrt{2} \times \ell(LK)$ ... (i)

and $\ell(OL) \ge \ell(LK)$ ... (ii)

From (i) and (ii) we can conclude that such a point can not be drawn.

Hence, [1].